Distribution estimators and confidence intervals for Cavalieri estimators

Johanna Ziegel, The University of Melbourne
joint work with Peter Hall, The University of Melbourne

Volume estimators based on Cavalieri's principle are widely used in the biosciences. For example in neuroscience, where volumetric measurements of brain structures are of interest, systematic samples of serial sections are obtained by magnetic resonance imaging or by a physical cutting procedure. The volume is then estimated by the sum over the areas of the structure of interest in the section planes multiplied by the width of the sections.

Assessing the precision of such volume estimates is a question of great practical importance, but statistically a challenging task due to the strong spatial dependence of the data and typically small sample sizes. The approach we take is more ambitious than earlier methodologies, the goal of which has been estimation of the variance of a volume estimator $\hat{v}$, rather than estimation of the distribution of $\hat{v}$; see e.g. Cruz-Orive (1999); Gundersen *et al.* (1999); García-Fiñana and Cruz-Orive (2004); Ziegel *et al.* (2010). We use a bootstrap method to obtain a consistent estimator of the distribution of $\hat{v}$ conditional on the observed data. Confidence intervals are then derived from the distribution estimate. We treat the case where serial sections are exactly periodic as well as when the physical cutting procedure introduces errors in the placement of the sampling points. To illustrate the performance of our method we conduct a simulation study with synthetic data and also apply our results to real data sets.

References

Cruz-Orive, L. M. (1999). Precision of Cavalieri sections and slices with local errors. *J. Microsc.*, **193**, 182–198.

García-Fiñana, M. and Cruz-Orive, L. M. (2004). Improved variance prediction for systematic sampling on $\mathbb{R}$. *Statistics*, **38(3)**, 243–272.

Gundersen, H. J. G., Jensen, E. B. V., Kiêu, K., and Nielsen, J. (1999). The efficiency of systematic sampling – reconsidered. *J. Microsc.*, **193**, 199–211.

Ziegel, J., Baddeley, A., Dorph-Petersen, K.-A., and Jensen, E. B. V. (2010). Systematic sampling with errors in sample locations. *Biometrika*, **97**, 1–13.